

0017-9310(94)E0090-H

Modified Zehner–Schlunder models for stagnant thermal conductivity of porous media

C. T. HSU

Department of Mechanical Engineering, The Hong Kong University of Science and Technology,
Clear Water Bay, Kowloon, Hong Kong

P. CHENG†

Department of Mechanical Engineering, University of Hawaii, Honolulu, HI 96822, U.S.A.

and

K. W. WONG

Department of Mechanical Engineering, The Hong Kong University of Science and Technology,
Clear Water Bay, Kowloon, Hong Kong

(Received 19 July 1993 and in final form 15 February 1994)

Abstract—Modified Zehner–Schlunder models are proposed for calculations of the stagnant thermal conductivity of two types of porous media with spatially periodic structures. The area-contact model is developed to take into consideration finite area contacts between spheres in a packed-sphere bed. It is shown that results based on the area-contact model are in better agreement with experimental data than those based on the original Zehner–Schlunder model especially at high solid/fluid thermal conductivity ratios. The phase-symmetry model is developed for a sponge-like porous medium where each phase is continuously connected and phase symmetry must be maintained in the expression of the stagnant thermal conductivity. The characteristics of these two models are illustrated.

1. INTRODUCTION

SINCE the end of the last century much theoretical work has been devoted to the study of the stagnant thermal conductivity of porous media. It is now recognized that the magnitude of the stagnant thermal conductivity of a porous medium depends not only on the porosity, the thermal conductivities of the solid and the fluid phases, but also on whether the particles are in good contact, point contact or no contact. Maxwell [1] obtained the first expression for the stagnant thermal conductivity of a packed-sphere bed by assuming a sufficiently dilute dispersion of spheres, which is valid as the limit of the porosity ϕ approaches unity ($\phi \rightarrow 1$). The expression given by Maxwell provides a lower bound for the stagnant thermal conductivity of a packed-sphere bed. Subsequently, numerous attempts have been made to extend Maxwell's solution to higher order of $(1-\phi)$. A review of these early particle non-touching models has been given by Churchill [2].

The first point-contact model for the stagnant thermal conductivity of a packed bed was given by Disseler and Eian [3] who chased a cube enclosing the spherical particle as a unit cell and assumed one-dimensional heat conduction in the cell. They assumed a simple cubic packing arrangement with six contact points between spheres. Dividing the unit cell into composite layers parallel to the heat flow direction, they obtained an analytical expression for the stagnant thermal conductivity of the bed. Unaware of Disseler and Eian's work, Kunii and Smith [4] obtained an approximate solution for the stagnant thermal conductivity of a packed-sphere bed. They chose a cube containing a spherical particle as a unit cell and assumed a minimum of two contact points between spheres, which were located on the two sides of the cube perpendicular to the heat flow direction. They assumed the heat conduction in the unit cell is through two parallel paths: the first path represents conduction through the gas-filled voids while the second path represents conduction through the solid and gas phases; the equivalent thermal resistance of the fluid and the solid phases in the second path is assumed to

† Author to whom correspondence should be addressed.

NOMENCLATURE

A_{fs}	solid and fluid interface	m	exponent in equations (9b) and (28)
B	shape factor in equations (6) and (10)	R	radius of the unit cell
b	shape factor in equation (22)	r_s	radius of the contact area
C	constant in equations (9b) and (28)	r, z	coordinates in Figs. 1 and 3
D_e	effective diffusivity in a porous medium	V_s	volume of the solid phase
D_f	diffusivity in a fluid phase	x, z	coordinates in Fig. 9.
F	volumetric fraction of the solid in the sandwiched layer shown in Fig. 3		
k_e	effective stagnant thermal conductivity of the saturated porous medium	Greek symbols	
k_f	thermal conductivity of the fluid phase	α	deformed factor
k_s	thermal conductivity of the solid phase	Λ	Wakao and Kato's contact area fraction
k_{sf}	equivalent thermal conductivity of a composite layer	λ	fluid/solid thermal conductivity ratio
		ϕ	porosity.

be in series. The fact that Kunii and Smith's expression compares well with their experimental data indicates that the thermal resistance in a packed-sphere bed can be considered as a combination of thermal resistances in parallel and in series. Using an approach similar to Kunii and Smith [4], Zehner and Schlunder [5] obtained a semi-empirical solution for the stagnant thermal conductivity of a packed bed, which has been widely used in the literature.

Both no-contact and point-contact models predict that the effective stagnant thermal conductivity of a porous medium is zero under vacuum conditions. The fact that finite but non-vanishing heat conduction takes place in a packed-sphere bed in vacuum is an indication that heat conduction through finite contact areas between spheres does exist. Ogniewicz and Yovanovich [6] as well as Chan and Tien [7] have analyzed the problem of constricted conduction through the contact surface of deformed spheres in an evacuated condition. It is pertinent to note that flattening of spheres in the vicinity of contact points may occur due to some external mechanical loads or to the weight of the bed itself. This heat conduction mechanism was neglected in the point-contact models because the thermal resistance due to point contact is infinite; consequently, heat conduction through this mechanism is vanishingly small and can be neglected under atmospheric pressure. The effect of finite area contacts between spheres on the stagnant thermal conductivity of an orthorhombic packing bed was investigated by Wakao and Kato [8] who used a finite-difference method to solve the conjugate heat conduction problem in the solid and the fluid phases within a unit cell. They found that the effect of finite-area contacts between spheres on the stagnant thermal conductivity is important at high solid/fluid conductivity ratio (k_s/k_f). A similar conclusion was reached later by Nozad *et al.* [9] who obtained the stagnant thermal conductivity of porous media composed of in-line cir-

cular and square cylinders (with and without interconnecting plates) by solving numerically the closure problems. The interconnecting plates represent area contacts between cylinders. If the contact parameter is equal to 0.01, they found that their numerical results matched their experimental data for a packed-sphere bed. Most recently, Hsu *et al.* [10] proposed a lumped-parameter model and obtained simple algebraic expressions for a number of periodic porous media composed of two- and three-dimensional particles. They found that their results based on in-line cubes with a touching parameter of 0.13 matched with Nozad *et al.*'s data [9] for the whole range of solid/fluid conductivity ratios.

In this paper, we shall modify Zehner and Schlunder's approach to obtain closed form solutions for the stagnant thermal conductivity of two types of spatially periodic porous media. The first model is for a packed-sphere bed with finite contact areas between spheres and the second model is for a sponge-like porous medium where each phase is continuously connected and phase-symmetry exists. It will be shown that the first model with finite contact areas taken into consideration is in better agreement with experimental data than that of the original Zehner-Schlunder model, especially at high solid/fluid conductivity ratios.

2. ZEHNER-SCHLUNDER'S MODEL

In this section, we shall briefly review the Zehner-Schlunder model. The unit cell for the Zehner-Schlunder model is shown in Fig. 1, that is an eighth of a cylinder with the shaded area denoting the solid phase. Zehner and Schlunder [5] assume that heat conduction in the unit cell follows two parallel paths: the first path is through the fluid in the outer concentric cylinder (of radii R and unity) and the second path is through the inner cylinder (of unit radius) which

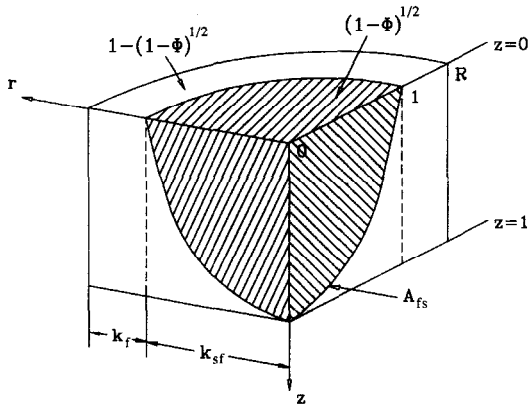


FIG. 1. Unit cell used by Zehner and Schlunder [5].

consists of both solid and fluid phases. Thus, the effective stagnant thermal conductivity k_e of the unit cell is given by

$$k_e = \left(1 - \frac{1}{R^2}\right)k_f + \frac{1}{R^2}k_{sf} \quad (1)$$

where k_{sf} is the equivalent thermal conductivity of the inner cylinder consisting of both fluid and solid phases and the value of R is to be determined. To determine the relationship between R and the porosity, Zehner and Schlunder [5] observe from mass transfer experiments that the diffusivity ratio of a fluid-saturated bed to that of a pure fluid is related to the porosity by

$$D_e/D_f = 1 - \sqrt{1 - \phi} \quad (2)$$

where D_e is the diffusivity of a fluid-saturated packed bed while D_f is the diffusivity of the fluid phase. Since heat conduction through a bed of non-conducting particles surrounded by a conducting gas is analogous to mass diffusion in a packed bed, they argue therefore that

$$\lim_{k_s/k_f \rightarrow 0} k_e/k_f = D_e/D_f = 1 - \sqrt{1 - \phi}. \quad (3)$$

If the thermal resistances of the solid and of the fluid in the composite layer are assumed in series, it follows that $k_{sf} \rightarrow k_s$ as $k_s/k_f \rightarrow 0$. At this limit, a comparison of equations (1) and (3) gives

$$\frac{1}{R^2} = \sqrt{1 - \phi}. \quad (4)$$

Substituting equation (4) into equation (1) gives

$$k_e = (1 - \sqrt{1 - \phi})k_f + \sqrt{1 - \phi}k_{sf}. \quad (5)$$

The value of k_{sf} depends on the shape of the solid and fluid interface A_{fs} (see Fig. 1) which is given by

$$r^2 + \frac{z^2}{[B - (B - 1)z]^2} = 1 \quad (6)$$

where B is the shape factor. Note that for $B = 0$ the boundary becomes the z -axis with no solid volume, at $B = 1$ the solid becomes a sphere, and at $B \rightarrow \infty$ the

solid occupies the entire inner cylinder. Assuming that the thermal resistances of the solid and fluid phases in the inner cylinder are in series with respect to the temperature gradient and with the aid of equation (6), Zehner and Schlunder [5] obtain the following expression for k_{sf} :

$$\frac{k_{sf}}{k_f} = \frac{2}{1 - \lambda B} \left(\frac{(1 - \lambda)B}{(1 - \lambda B)^2} \ln \frac{1}{\lambda B} - \frac{B + 1}{2} - \frac{B - 1}{1 - \lambda B} \right) \quad (7)$$

where $\lambda = k_f/k_s$. Substituting equation (7) into equation (5) yields

$$\frac{k_e}{k_f} = 1 - \sqrt{1 - \phi} + \frac{2\sqrt{1 - \phi}}{1 - \lambda B} \left(\frac{(1 - \lambda)B}{(1 - \lambda B)^2} \ln \frac{1}{\lambda B} - \frac{B + 1}{2} - \frac{B - 1}{1 - \lambda B} \right). \quad (8)$$

The shape factor B in equation (8) is determined from a geometric condition which gives

$$\phi = 1 - \left(\frac{B(3 - 4B + B^2 + 2 \ln B)}{(B - 1)^3} \right)^2. \quad (9a)$$

The above equation is approximated by

$$B = C \left(\frac{1 - \phi}{\phi} \right)^m. \quad (9b)$$

Zehner and Schlunder [5] found that equation (9b) with $C = 1.25$ and $m = 10/9$ matched with the exact expression given by equation (9a). Based on a least-square procedure, however, we found that equation (9b) with $C = 1.364$ and $m = 1.055$ would match better with equation (9a).

The stagnant thermal conductivity given by equations (8) and (9b) as a function of k_s/k_f for different values of porosity are presented in Fig. 2. For comparison, experimental data obtained by Nozad *et al.* [9] are also plotted in the same graph. It is noted from this graph that the Zehner-Schlunder model with $\phi = 0.42$ is in good agreement with experimental data for $0 < k_s/k_f < 10^3$. For higher value of the thermal conductivity ratio ($k_s/k_f > 10^3$), it is shown that the Zehner-Schlunder model underpredicts the stagnant thermal conductivity as was pointed out previously by Kaviany [11].

3. AREA-CONTACT MODEL

It can be postulated that the reason the Zehner-Schlunder model underpredicts the stagnant thermal conductivity of a packed-sphere bed at high solid/fluid conductivity ratios is owing to that fact that the model assumes point contacts between spheres. As mentioned earlier, finite-area contacts between spheres are likely to occur in a packed-sphere bed because of external loads or owing to their own weight. To take into consideration the finite-area contact between spheres, we now modify the solid/fluid interface by the following equation:

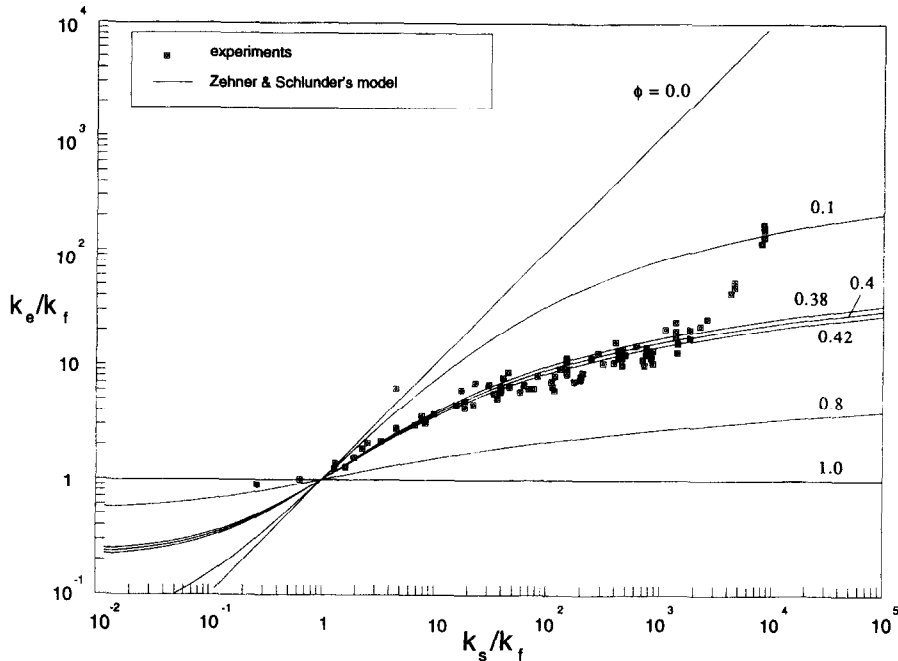


FIG. 2. Comparison of Zehner-Schlunder model [5] with Nozad *et al.*'s data [8].

$$r^2 + \frac{z^2}{[(1+\alpha)B - (B-1)z]^2} = 1 \quad (10)$$

where α is the deformed factor. The modified solid/fluid interface A_{fs} is shown in Fig. 3 where r_s is the radius of the contact area that can be determined by imposing the condition: $r = r_s, z = 1$. It follows from equation (10) that

$$r_s^2 = 1 - \frac{1}{(1+\alpha B)^2} \quad (11)$$

If the one-dimensional heat conduction in the unit cell (shown in Fig. 3) consists of three parallel paths: the outer concentric cylinder (of radii R and unity) consisting of the fluid phase, the middle concentric cylinder (of radii unity and r_s) consisting of both solid and fluid phases, and the inner cylinder (of radius r_s) consisting of the solid phase, the effective stagnant

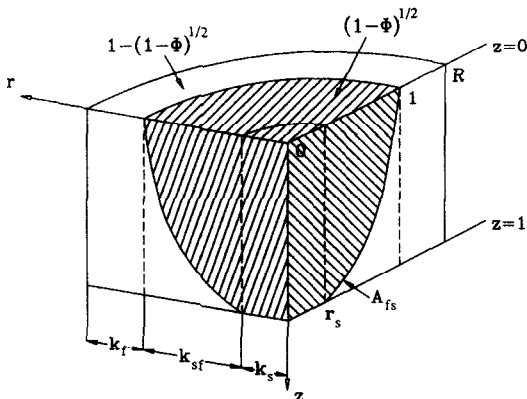


FIG. 3. Unit cell for the area-contact model.

thermal conductivity is given by

$$k_e = \left(1 - \frac{1}{R^2}\right)k_f + \left(\frac{1-r_s^2}{R^2}\right)k_{fs} + \left(\frac{r_s}{R}\right)^2 k_s \quad (12)$$

where R is given by equation (4). In the above equation, k_{fs} is the equivalent thermal conductivity of the middle composite concentric cylinder that can be determined from

$$(\pi - \pi r_s^2)k_{sf} = \int_{r_s}^1 2\pi r k^*(r) dr \quad (13a)$$

where

$$\frac{k^*(r)}{k_f} = \frac{1}{(1-z) + \lambda z} \quad (13b)$$

which is obtained from the layer-in-series method. Substituting equation (13b) into equation (13a) and with the aid of equations (10) and (11), the resulting equation can be rewritten as

$$\frac{k_{sf}}{k_f} = 2(1+\alpha B)^2 \int_0^1 \frac{(1+\alpha)Bz dz}{[1 + (\lambda-1)z][(1+\alpha)B - (B-1)z]^3} \quad (14)$$

The above integral can be integrated to give

$$\begin{aligned} \frac{k_{sf}}{k_f} = & \frac{2}{[1 - \lambda B + (1-\lambda)\alpha B]} \left(\frac{(1-\lambda)(1+\alpha)B(1+\alpha B)^2}{[1 - \lambda B + (1-\lambda)\alpha B]^2} \right. \\ & \left. \times \ln \frac{1+\alpha B}{(1+\alpha)B\lambda} - \frac{B+1+2\alpha B}{2} - \frac{(B-1)(1+\alpha B)}{1 - \lambda B + (1-\lambda)\alpha B} \right) \end{aligned} \quad (15)$$

Substituting equation (15) into equation (12) yields

$$\begin{aligned} \frac{k_e}{k_f} = & [1 - \sqrt{(1-\phi)}] + \frac{\sqrt{(1-\phi)}}{\lambda} \left(1 - \frac{1}{(1+\alpha B)^2} \right) \\ & + \frac{2\sqrt{(1-\phi)}}{[1-\lambda B + (1-\lambda)\alpha B]} \left(\frac{(1-\lambda)(1+\alpha)B}{[1-\lambda B + (1-\lambda)\alpha B]^2} \right. \\ & \times \ln \frac{1+\alpha B}{(1+\alpha)B\lambda} - \frac{B+1+2\alpha B}{2(1+\alpha B)^2} \\ & \left. - \frac{(B-1)}{[1-\lambda B + (1-\lambda)\alpha B](1+\alpha B)} \right) \end{aligned} \quad (16)$$

which is the effective stagnant thermal conductivity of a packed-sphere bed with finite contact area of the spheres taken into consideration. Note that equation (16) with $\alpha = 0$ (no deformation) reduces to equation (8) given by Zehner and Schlunder [5].

To determine the value of B , we note that the volume fraction of the solid in the unit cell is

$$1 - \phi = \frac{V_s}{\pi R^2} \quad (17)$$

where V_s is the volume of the solid which can be obtained from

$$\begin{aligned} V_s = & \pi \int_0^1 \left(1 - \frac{z^2}{[(1+\alpha)B - (B-1)z]^2} \right) dz \\ = & \frac{\pi B}{(B-1)^2} \left(\frac{2(1+\alpha)}{B-1} \ln \frac{(1+\alpha)B}{1+\alpha B} \right. \\ & \left. + \frac{(B-3) + \alpha(B^2 - 2B - 1)}{1+\alpha B} \right) \end{aligned} \quad (18)$$

Substituting equation (18) into equation (17) and with the aid of equation (4) yields

$$\begin{aligned} \phi = & 1 - \frac{B^2}{(1-B)^6(1+\alpha B)^2} \left[(B^2 - 4B + 3) \right. \\ & \left. + 2(1+\alpha)(1+\alpha B) \ln \frac{(1+\alpha)B}{1+\alpha B} + \alpha(B-1)(B^2 - 2B - 1) \right]^2 \end{aligned} \quad (19)$$

which reduces to equation (9a) if $\alpha = 0$. For given values of α and ϕ , equation (19) can be solved for B by iterations. However, it can also be approximated by equation (9b) with C and m being a function of α . The values of C and m for different values of α (as determined by a least-square fit) are presented in Fig. 4. The values of B versus ϕ for a specific value of α ($\alpha = 0.002$) based on equation (19) and the approximate expression of equation (9b) are plotted in Fig. 5 for comparison purposes. It is seen that they are in excellent agreement.

Computations were carried out for equation (16) with $\phi = 0.42$ for various values of α . It was found that equation (16) with $\phi = 0.42$ and $\alpha = 0.002$ would match with Nozad *et al.*'s data at high solid/fluid conductivity ratio as shown in Fig. 6. The results

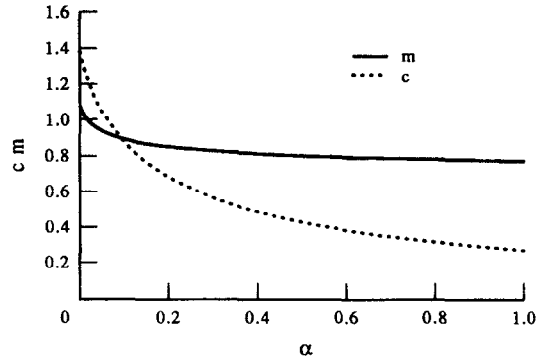


FIG. 4. Variations of C and m with deformed factor α .

based on Nozad *et al.*'s two-dimensional numerical model [8] and Hsu *et al.*'s three-dimensional lumped parameter model [10] are also presented in the same plot for comparison purposes. It is shown that results based on equation (16) are slightly lower than Nozad *et al.*'s numerical solution and Hsu *et al.*'s model for $k_s/k_f > 10^3$. It will be instructive to present the results of Fig. 6 in terms of the contact area radius r_s instead of the deformed parameter α . For given values of ϕ and α , B can be determined either from equation (19) or from equation (9b) and Fig. 4, and consequently r_s can be determined. The results of these computations are presented in Fig. 7 where $r_s = 0.0816$ corresponds to $\alpha = 0.002$ and $\phi = 0.42$. Since the Wakao and Kato's contact area fraction Λ is related to the radius of the contact area r_s by the relation $\Lambda = r_s^2$, it follows that the value of $r_s = 0.0861$ corresponds to the contact parameter of $\Lambda = 0.0074$. A similar plot for the dimensionless stagnant thermal conductivity versus thermal conductivity ratio at selected values of the contact area fraction for an orthorhombic packing (with $\phi = 0.259$) was presented by Wakao and Kato [8] based on a finite-difference solution. A comparison of results of Fig. 7 and those obtained by Wakao and Kato [8] shows that they are in excellent agreement for $k_s/k_f < 100$. The results based on the present model become higher than those of Wakao and Kato when $k_s/k_f > 100$. For example, at $k_s/k_f = 10^4$ and $r_s = 0.0861$ (i.e. $\Lambda = 0.0074$) our model gives

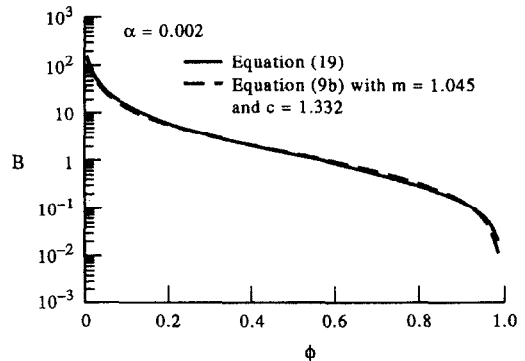


FIG. 5. Exact and approximate values of the shape factor for the area-contact model.

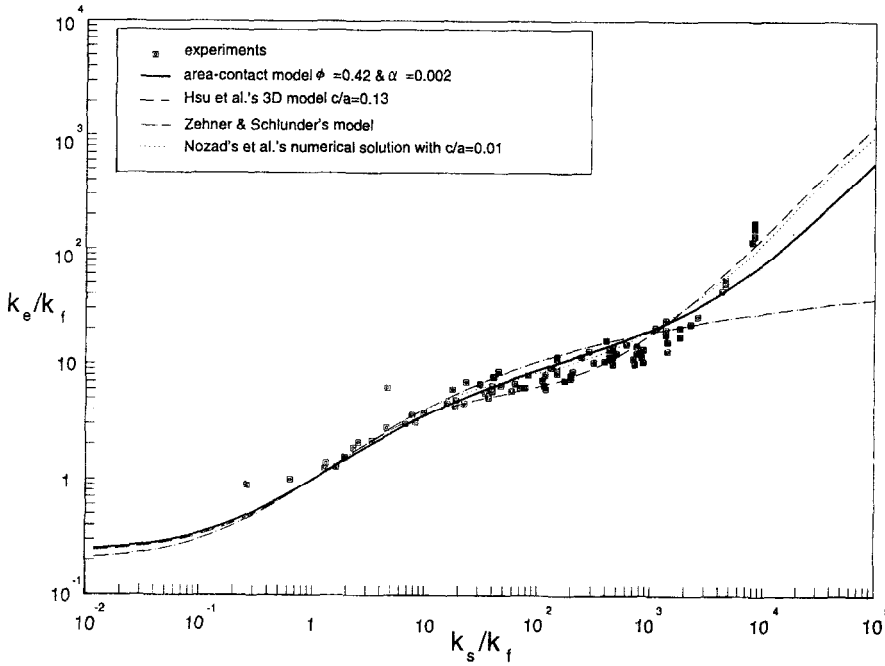


FIG. 6. Comparison of the area-contact model with other theoretical models and experimental data.

$k_e/k_f = 70$ while Wakao and Kato's model gives $k_e/k_f = 30$. Thus, our results are closer to the limiting case of $k_s/k_f \rightarrow \infty$, $k_e/k_f \rightarrow \Lambda k_s/k_f$. The inconsistency of Wakao and Kato's results with the limiting case may be attributed to insufficient resolution of grid size used in the numerical solution. Figure 8 shows the effect of porosity on the stagnant thermal conductivity given by the point-contact model and the area-contact model, i.e. equation (16) with $\alpha = 0$ and 0.002 respec-

tively. It can be seen that the finite touching area has an important effect on the effective stagnant thermal conductivity only for $k_s/k_f > 10^3$.

4. PHASE-SYMMETRY MODEL

We now consider a sponge-like porous medium such that each phase is continuously connected and has a phase symmetry. Equations (8) and (16) cannot

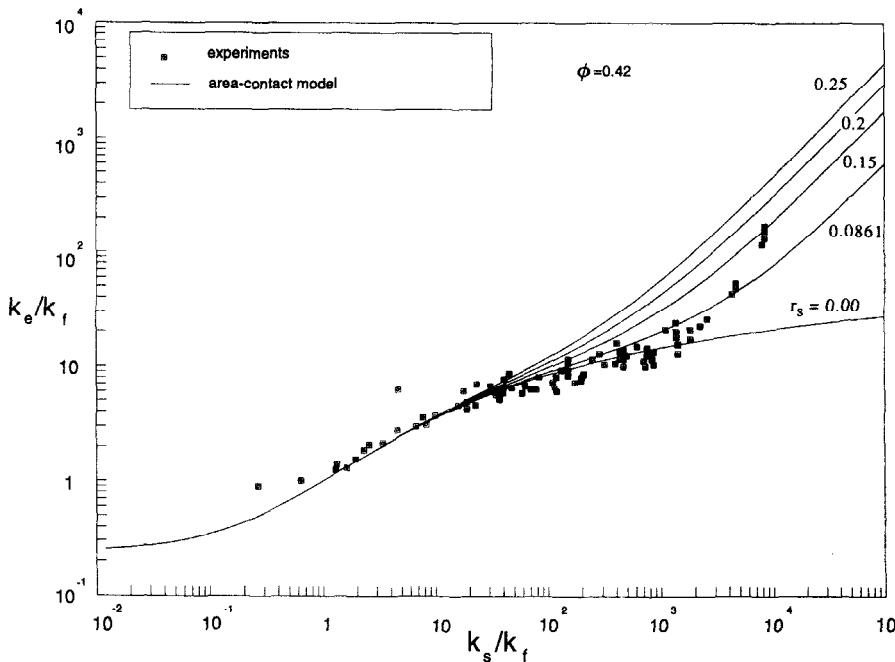


FIG. 7. Effects of contact-area radius on the stagnant thermal conductivity.

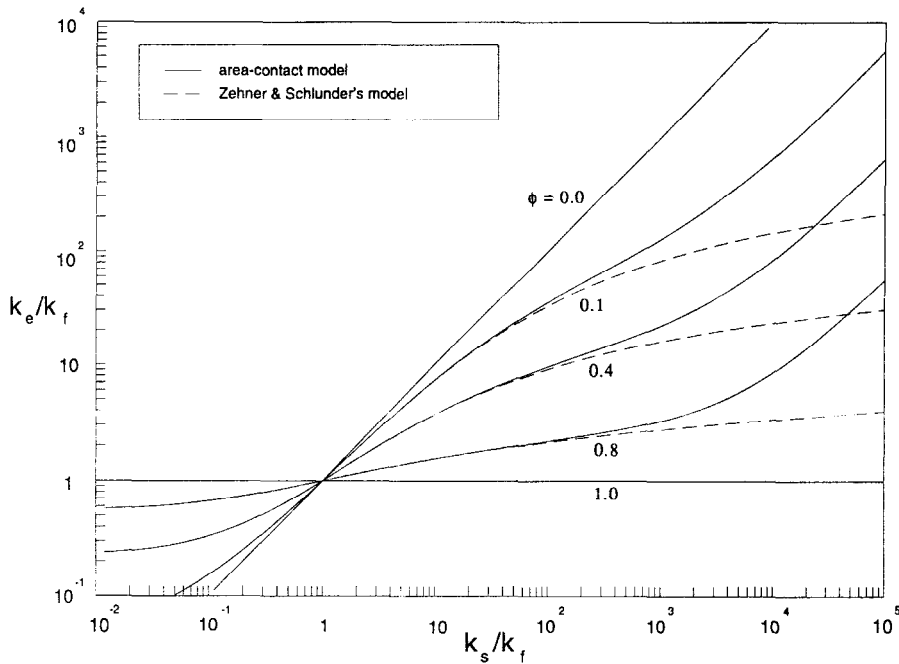


FIG. 8. Effect of porosity on the stagnant thermal conductivity of the area-contact model.

be applied to this type of porous medium because these expressions are not symmetric with respect to the two phases. To derive an expression for the stagnant thermal conductivity of this type of porous medium, consider a unit cell as sketched in Fig. 9. The unit cell consists of three parallel layers: a fluid layer and a solid layer with a middle composite layer to account for the interfacial effect. In symmetry with equation (3), we require that

$$\lim_{k_f/k_s \rightarrow 0} k_e/k_s = 1 - \sqrt{\phi}. \quad (20)$$

Therefore, the effective thermal conductivity of the

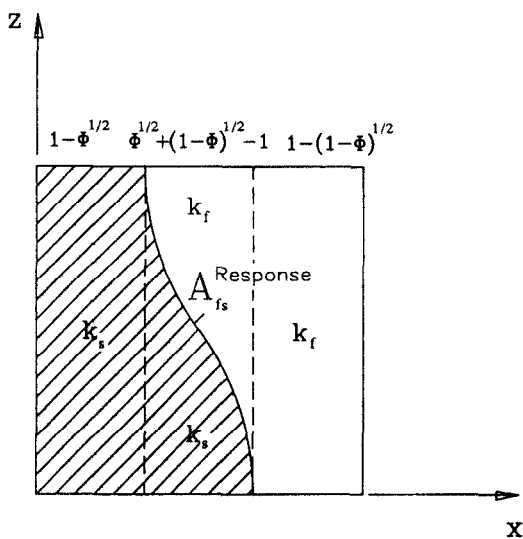


FIG. 9. Unit cell for the phase-symmetry model.

saturated porous medium consisting of the three parallel layers is

$$k_e = [1 - \sqrt{(1 - \phi)}]k_f + (1 - \sqrt{\phi})k_s + [\sqrt{(1 - \phi)} + \sqrt{\phi - 1}]k_{sf} \quad (21)$$

where k_{sf} is the equivalent thermal conductivity of the composite layer. The solid-fluid interface is given by

$$x + \frac{z}{b - (b - 1)z} = 1 \quad (22)$$

where b is the shape factor to be determined. The value of k_{sf} can be computed from the layer-in-series model which gives

$$\frac{k_{sf}}{k_f} = \frac{b(1 - \lambda)}{(1 - \lambda b)^2} \ln \frac{1}{\lambda b} - \frac{b - 1}{1 - \lambda b}. \quad (23)$$

Substituting equation (23) in equation (21) leads to

$$\begin{aligned} \frac{k_e}{k_f} = & [1 - \sqrt{(1 - \phi)}] + \frac{(1 - \sqrt{\phi})}{\lambda} \\ & + [\sqrt{(1 - \phi)} + \sqrt{\phi - 1}] \left(\frac{b(1 - \lambda)}{(1 - \lambda b)^2} \ln \frac{1}{\lambda b} - \frac{b - 1}{1 - \lambda b} \right) \end{aligned} \quad (24a)$$

which in the limit of $\lambda b \rightarrow 1$ reduces to

$$\begin{aligned} \frac{k_e}{k_f} = & [1 - \sqrt{(1 - \phi)}] + (1 - \sqrt{\phi})/\lambda \\ & + [\sqrt{(1 - \phi)} + \sqrt{\phi - 1}](b + 1)/2. \end{aligned} \quad (24b)$$

Note that equation (24) is symmetric with respect to the conductivity ratio as well as the volumetric ratio of the fluid and solid phases.

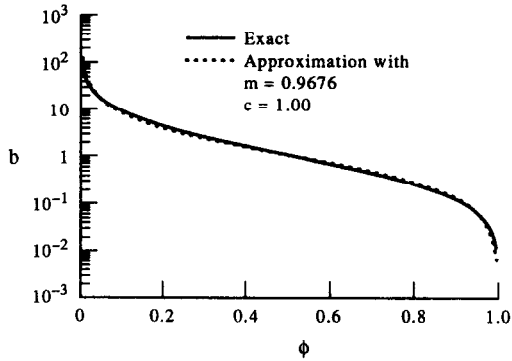


FIG. 10. Exact and approximate evaluations of the shape factor for the phase-symmetry model.

The value of the shape factor b as a function of the porosity for the phase-symmetry model can be determined as follows. The volumetric fraction of the solid in the sandwiched layer is given by

$$F = \frac{\sqrt{\phi - \phi}}{\sqrt{(1 - \phi) + \sqrt{\phi - 1}}} \quad (25)$$

On the other hand, the value of F can be calculated from

$$F = \int_0^1 \frac{bz \, dz}{[b - (b - 1)z]^2} = \frac{b}{(b - 1)^2} (b - 1 - \ln b) \quad (26)$$

Equating equations (25) and (26) gives

$$\frac{\sqrt{\phi - \phi}}{\sqrt{(1 - \phi) + \sqrt{\phi - 1}}} = \frac{b}{(1 - b)^2} (b - 1 - \ln b) \quad (27)$$

which is the relationship between ϕ and b . For a given value of ϕ , equation (27) can be solved numerically

for b by iteration. As before, however, a good approximation is given by

$$b = C \left(\frac{1 - \phi}{\phi} \right)^m \quad (28)$$

It is found that equation (28) with $m = 0.9676$ and $C = 1$ gives results which match those based on equation (27), as is shown in Fig. 10. Note that the form of equation (28) was chosen such that it satisfies the same constraints as equation (27) at the limits of $\phi = 0$ and 1 where $b = \infty$ and 0, respectively. If C is taken to be 1, then the constraint at $\phi = 0.5$ where $b = 1$ is also satisfied. The effect of porosity on the phase symmetry model is presented in Fig. 11. A comparison of Figs. 8 and 11, at given values of k_s/k_f and ϕ , shows that the stagnant thermal conductivity given by the phase-symmetry model is higher than those based on the area-contact model.

5. CONCLUDING REMARKS

Analytical expressions for the stagnant thermal conductivity of two types of spatially periodic porous media are obtained based on modifications of the well-known Zehner-Schlunder model. The first model takes into consideration the finite contact areas between spheres which is important for $k_s/k_f > 10^3$ in a packed-sphere bed. With a deformed factor $\alpha = 0.002$ and porosity $\phi = 0.42$, the area-contact model is in excellent agreement with experimental data of a packed-sphere bed for a wide range of solid/fluid thermal conductivity ratios. The second model is for a porous medium with phase symmetry that will have important applications for composite materials. The

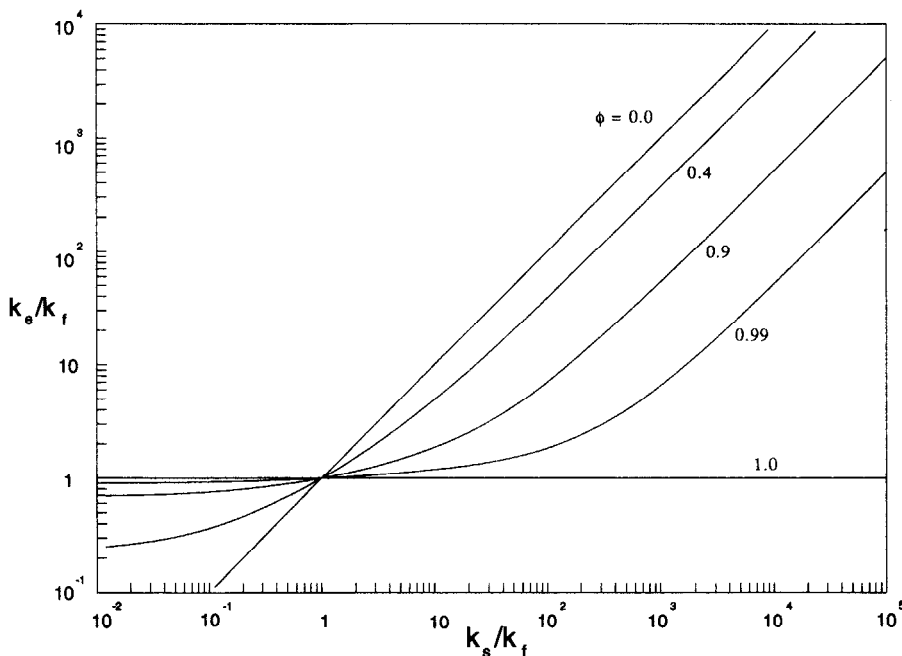


FIG. 11. Effect of porosity on the phase-symmetry model.

accuracy of the phase-symmetry model cannot be assessed at this time because of the lack of experimental data.

REFERENCES

1. J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Vol. 1, p. 365 (1873).
2. S. W. Churchill, The thermal conductivity of dispersions and packed beds—an illustration of the unexploited potential of limiting solutions for correlation, *Adv. Transport Processes* **4**, 394–418 (1986).
3. R. D. Disseler and C. S. Eian, Investigation of effective thermal conductivities of powders, NACA RM E52C05 (1952).
4. D. Kunij and J. M. Smith, Heat transfer characteristics of porous rocks, *A.I.Ch.E. Jl* **6**, 71–78 (1960).
5. P. Zehner and E. U. Schlunder, Thermal conductivity of granular materials at moderate temperatures (in German), *Chemie. Ingr.-Tech.* **42**, 933–941 (1970).
6. Y. Ogniewicz and M. M. Yovanovich, *Effective Conductivity of Regularly Packed-Spheres: Basic Cell Model with Constriction, Heat Transfer and Thermal Control Systems (Progress in Astronautics and Aeronautics Vol. 60)* (Edited by L. S. Fletcher). American Institute of Aeronautics and Astronautics, Inc., Washington, DC (1978).
7. C. K. Chan and C. L. Tien, Conductance of packed pheres in vacuum, *Trans. ASME*, August 302–308 (1973).
8. N. Wakao and K. Kato, Effective conductivity of packed beds, *J. Chem. Engng Jap.* **2**, 24–32 (1969).
9. S. Nozad, R. G. Carbonell and S. Whitaker, Heat conduction in multiphase systems. I: theory and experiments for two-phase systems, *Chem. Engng Sci.* **40**, 843–855 (1985).
10. C. T. Hsu, P. Cheng and K. W. Wong, A lumped-parameter model for stagnant thermal conductivity of spatially periodic porous media, submitted for publication (1993).
11. M. Kaviany, *Principles of Heat Transfer in Porous Media*. Springer-Verlag, New York (1991).